

# Asset Prices and Ambiguity<sup>†</sup>

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November 18, 2011

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<sup>†</sup> We benefited from discussions with Avi Wohl, Azi Ben-Refael, Rob Engel, Saggi Katz, Ilan Cooper, Yakov Amihud, Marti Subrahmanyam, Itzhak Gilboa, David Schmeidler, Sergiu Hart, Adam Brandenburger, Eitan Goldman, Jacob Oded, Jacob Boudoukh, Ilan Kremer, Doron Avramov and seminar participants at New York University, the Interdisciplinary Center (IDC), Herzliya and The Hebrew University of Jerusalem.

# ASSET PRICES AND AMBIGUITY

## Abstract

Modern portfolio theory, developed in the expected utility paradigm, focuses on the relationship between risk and return, assuming away ambiguity, uncertainty over the probability space. In this paper, we present an asset pricing model developed by Izhakian (2011), which incorporates ambiguity as a second factor (in addition to “risk”). Our contribution is two fold; we propose an ambiguity measure that is derived theoretically and computed from intra-day stock market prices. Second, we use it in conjunction with risk measures to test the basic relationship between risk, ambiguity and return. We find that our ambiguity measure has a consistently negative effect on returns and that our risk measure has mostly a positive effect. The best evidence, judging by statistical significance, is obtained when we use the change in volatility alongside the measure of ambiguity.

**Key words:** Asset pricing, Ambiguity, Ambiguity measurement.

**JEL codes:** C44, C65, D81, D83, G11, G12.

# 1 Introduction

The fundamental relationship between risk and return of the market portfolio in the mean-variance paradigm is given by the following equation

$$E[r_m] - r_f = \gamma(\sigma_m^2), \quad (1)$$

where  $r_m$  is the return on the market portfolio,  $r_f$  is the risk free rate,  $\sigma_m^2$  is the risk of the market portfolio and  $\gamma(\cdot)$  is a measure of risk aversion of a representative agent (or, an aggregation of risk aversion coefficients of investors). This linear relationship has been subjected to several time series empirical tests. Merton (1980) and French, Schwert and Stambaugh (1987) are two classic examples of such tests. In general, the tests have low  $R^2$  and some of these tests produce negative coefficients of absolute risk aversion.

We believe that a missing factor that determines the equity premium is ambiguity<sup>1</sup> and the attitude towards it. Though there is an abundance of research on various aspects of ambiguity and ambiguity aversion, there is almost no empirical work providing a measure of ambiguity and incorporating such a measure in tests of the relationship between risk and return.

In this paper we introduce a measure of ambiguity, which is an additional factor determining the equity premium. Equation (2) below is the expanded version of Equation (1) incorporating ambiguity. That is,

$$E[r_m] - r_f = \gamma(\sigma_m^2) + \eta(\mathcal{U}^2), \quad (2)$$

where  $\mathcal{U}^2$  measures the degree of ambiguity and  $\eta(\cdot)$  is a measure of investors' attitude toward ambiguity. This measure is an outcome of the theoretical model developed by Izhakian (2011). The results are highly significant, challenging the conventional wisdom on investors' attitudes towards ambiguity. To the best of our knowledge, our study is the first empirical study that uses market data to measure ambiguity based on a theoretically derived model that combines risk and ambiguity.

Modern portfolio theory, until recently, has practically ignored the Knightian distinction between risk and uncertainty. There were some exceptions like the collection of papers in a book

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<sup>1</sup> The economic literature has been using the word ambiguity to describe Knightian uncertainty.

edited by Bawa and Brown and Klien (1979). These papers, however, focus on estimation risk, how to correct for it, and how to incorporate it in portfolio selection or how it may affect capital market equilibrium<sup>2</sup>. They did not deal with ambiguity and how it may affect asset prices, i.e., the relationship between ambiguity and return. Should ambiguity be priced? Can we separate risk and risk attitudes from ambiguity and attitudes toward ambiguity? How can we measure ambiguity? These are questions that, to the best of our knowledge, have not been answered rigorously.

The paper by Izhakian (2011) provides the theoretical underpinning of our paper which focuses on issues of ambiguity measurement and tests of risk-ambiguity-return relationships. In his paper Izhakian (2011) introduced a novel model of ambiguity, called *Shadow probability theory* (henceforth *Shadow Theory*) and studied how it affects investors' choices. The model provides a measure for the degree of ambiguity which is the center piece of the empirical tests that we employ in this paper. We focus on testing the effect of ambiguity on asset prices in a time series context while using the S&P500 index as the market portfolio. Our empirical results show that this measure has a significant effect on stock market returns.

We assume a representative investor whose reference point is 0 excess-return. Assets' excess-returns are classified as gains or losses. Excess returns lower than 0 are considered as a loss and returns equal or higher than 0 are considered as gains. All assets' excess returns are assumed to be normally distributed. However, the parameters governing the distributions, i.e., mean and variance, are unknown and assumed to be random with a uniform distribution.<sup>3</sup>

We show that, ambiguity and the return on the market portfolio are negatively correlated which implies that the degree of ambiguity is taken into account by investors when they price financial assets. It also implies that the representative investor, who holds the market portfolio exhibits ambiguity loving.

What is the evidence regarding investor's attitude toward ambiguity? It turns out that it depends on the states of nature that the investor faces. If he is faced with a high probability of

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<sup>2</sup> In recent years there is a surge in papers that try to incorporate Knightian uncertainty naming it 'model risk'. For example, Uppal and Wang (2003), Epstein and Schneider (2008), Dreschsler (2010), Ju and Miao (2011) and , Anderson Ghysels and Juergens (2009)). Some of them just do calibrations while others use proxies like disagreement among analysts, etc.

<sup>3</sup> Since there is no information on the likelihood of any of the alternatives it is reasonable to assume equal probability.

losses he tends to embrace ambiguity, while if he faces a high probability of gains he may be ambiguity averse.

Viscusi and Chesson (1999) found that people exhibit ‘fear’ effects of ambiguity for small probabilities of suffering a loss and ‘hope’ effects for large probabilities of loss.<sup>4</sup> Considering investors in the stock market, where the probability of loss is relatively high (almost 50%), one would expect to observe ambiguity loving. Ivanov (2011) shows that more individuals exhibit ambiguity loving than ambiguity aversion. In particular, 32% are classified as ambiguity-loving, comparing to only 22% who are classified as ambiguity aversion; the remaining 46% are considered as ambiguity neutral. Assuming risk neutrality, Maffioletti and Michele (2005) also found ambiguity seeking in individuals' trading behavior. Analyzing statistical information of probabilities about health insurance, Wakker, Timmermans and Machielse (2007) document that individuals are ambiguity seeking. In an experimental study of bidders' behavior Chen, Katuščák and Ozdenoren (2007) suggests that individuals are ambiguity seeking. In general, most behavioral studies find ambiguity loving behavior when there is some chance of a loss.

Consistent with the above studies, our results show that investors are ambiguity lovers. In our study the average probability of loss is relatively high (almost 50%) and as found by Viscusi and Chesson (1999), in such cases, investors are ambiguity loving. These findings are consistent with our theoretical model. When returns are symmetrically distributed, an investor who maximizes expected return minimizes the probability of loss. Therefore, when the probability of loss is relatively high, given two assets with identical risk, he prefers the asset with the random probability over an asset with known probabilities. Such preferences imply ambiguity loving.

The rest of the paper is organized as follow. Section 2 establishes the theoretical framework. Section 3 discusses the data and the empirical methodology. Section 4 presents the empirical findings and Section 5 provides summary and conclusions.

## **2 The theoretical model**

Recently, Izhakian (2011) introduced a novel model of ambiguity, called *Shadow theory*, which provides a measure of the degree of ambiguity. This measure is the center piece of the

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<sup>4</sup> Abdellaoui, Baillon, Placido and Wakker (2011) tie ambiguity loving to the source of ambiguity.

empirical tests that are conducted in this paper. In the next couple of pages we provide a detailed summary of the main principles of Shadow theory and how we used it in the empirical tests.

## **2.1 Preliminaries**

The theory of financial assets prices is mainly based on the expected utility (von Neumann-Morgenstern (1944) and Savage (1961)) paradigm, which assumes that decision makers know, or act as if they know, the probabilities of all states of nature. A basic issue with these models is that in reality the investor does not know the precise probabilities of events (see Ellsberg (1961)), which means that individuals are exposed not only to *risk* but also to *ambiguity* (*Knightian uncertainty*). Using two main approaches, several models have been suggested to address decision making under uncertainty. Subjective nonadditive probabilities of Gilboa (1987) and the Choquet expected utility (CEU) of Schmeidler (1989) state that beliefs can be represented by a single, but nonadditive, prior. The multiple prior (MEU) approach of Gilboa and Schmeidler (1989) does not assume that the decision maker's belief can be represented by a single additive prior and instead assumes a multiple prior representation. The multiple prior approach was extended to the model misspecification approach (Hansen and Sargent (2001)) and non-reducible second-order probabilities approach (Segal (1987) and Klibanoff, Marinacci, and Mukerji (2005)).<sup>5</sup> While this literature made a considerable contribution to understanding the decision maker's preferences toward ambiguity, a complete separation between ambiguity and risk, which enables to measure ambiguity empirically, has not been derived. Such a measure is necessary in testing the effect of ambiguity on asset prices.

*Shadow Theory* assumes that probabilities (capacities) of observable events and their outcomes, in an outcome-space, are random and are dominated by unobserved events in a latent directing-space, with a second-order probability.<sup>6</sup> In this framework a complete separation between risk and ambiguity and between preferences and beliefs is obtained. This allows us to measure the level of ambiguity. In this model, ambiguous probabilities are subjectively interpreted by decision makers in a nonlinear way, characterized by probabilistic sensitivity to ambiguity. Ambiguity aversion, thus, takes the form of sub-additive probabilities (i.e., the

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<sup>5</sup> Other models that relax the reduction between first and second order probabilities include Klibanoff et al. (2009), Ju and Miao (2011), Hayashi and Miao (2011), Ergin and Gul (2009)[30], Nau (2006)[75], and Chew and Sagi (2008)[18].

<sup>6</sup> Capacities refer to subjective probabilities which are not necessarily additive.

probabilities add up to number smaller than 1). In this context, when ambiguity is present, ambiguity loving implies a super-additive (subjective) probability measure (i.e., the probabilities add up to number greater than 1).

The Shadow theory developed in Izhakian (2011) extends the Choquet expected utility of Schmeidler (1989) and Tversky and Kahneman's (1992) cumulative prospect theory.<sup>7</sup> It adds reference-dependent beliefs and applies a two-side Choquet expected utility for losses and for gains, separately. Shadow theory assumes that the financial decision maker (henceforth DM or investor) has a reference point that separates losses from gains. Outcomes are classified as a gain or a loss relative to the selected reference point. Outcomes that are lower than the subjective reference point are considered a loss and outcomes which are higher than the reference points are considered a gain. Unlike the original prospect theory, which assumes a reference-dependent preferences and a DM who exhibits loss aversion, Izhakian (2011) assumes reference-dependent beliefs and a DM who does not exhibits loss aversion but exhibits ambiguity aversion. The reference point in Izhakian (2011) serves as the reference that separates the subjective probabilities of gains from the subjective probabilities of losses. The volatility of these probabilities serves for measuring the level of ambiguity. Attitude toward ambiguity is formed with respect to this level of ambiguity.

The implication of a sub-additive probability measure for asset prices is that there is an ambiguity premium in addition to the conventional risk premium. The conventional *risk premium* is the premium that a DM is willing to pay for replacing a risky bet by its expected outcome. The *ambiguity premium* is the premium that a DM is willing to pay for replacing an ambiguous bet by a risky, non ambiguous, bet with an identical expected outcome. The *uncertainty premium* is the total premium that a DM is willing to pay for replacing an ambiguous bet by its expected outcome, i.e., it contains both, a risk premium and an ambiguity premium.

The uncertainty premium is provided by

$$K = -\frac{1}{2} \frac{U''(E[x])}{U'(E[x])} \text{Var}[x] - \frac{1}{4} \left[ \frac{1}{2} \frac{\psi''(E[P_L])}{\psi'(E[P_L])} + \frac{1}{2} \frac{\psi''(E[P_G])}{\psi'(E[P_G])} \right] E[x] U^2[x], \quad (3)$$

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<sup>7</sup> Cumulative prospect theory, developed by Tversky and Kahneman (1992), generalizes the original prospect theory, Kahneman and Tversky (1979). It replaces risk by uncertainty and modifies the probability weights to allow a space with an infinite support and to deal with issues related to stochastic dominance.

where

$$R = -\frac{1}{2} \frac{U''(E[x])}{U'(E[x])} \text{Var}[x]$$

is the risk premium and the second component,

$$A = -\frac{1}{4} \left[ \frac{1}{2} \frac{\psi''(E[P_L])}{\psi'(E[P_L])} + \frac{1}{2} \frac{\psi''(E[P_G])}{\psi'(E[P_G])} \right] E[x] \mathcal{U}^2[x]$$

is the ambiguity premium. The parameters  $P_L$  and  $P_G$  are the random probabilities of loss and of gain, respectively. The expectation  $E[P_L]$  and  $E[P_G]$  are taken with respect to the likelihoods of the possible probability measures; i.e., with respect to the second-order probabilities. That is,

$$E[P_L] = \frac{1}{M} \sum_{i=1}^M P_i(x < x_k) \quad (4)$$

and

$$E[P_G] = \frac{1}{M} \sum_{i=1}^M P_i(x \geq x_k), \quad (5)$$

where  $x_k$  the reference outcome which distinguishes losses and gains.

The expected outcome is  $E[x]$ , where the expectation of the outcome is evaluated using the expected probabilities for each outcome. It combines two expectations; with respect to the random outcomes and with respect to the random probabilities. The parameter

$$\mathcal{U}^2[x] = 4 \text{Var}[P_L] \quad (6)$$

is Izhakian's measure of ambiguity, which is four times the variance of the probability of loss or four times the variance of the probability of gain.<sup>8</sup> It is important to note that,  $\mathcal{U}^2 \in [0,1]$ , attains its minimum value, 0, when all probabilities are known, and its maximum value, 1, only in the extreme case of binomial distribution with a random probability for each event that can have

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<sup>8</sup> This equality is obtained since the variance the probability of event is equals to the variance of the probability of its complement event.

probabilities of 0 or 1 with equal chances. The Arrow-Pratt coefficient of absolute risk aversion is  $-\frac{U''(\cdot)}{U'(\cdot)}$ . The coefficient of absolute ambiguity aversion is  $-\frac{\psi''(\cdot)}{\psi'(\cdot)}$ .

Equation (3) defines the premiums required by investors for bearing the risk and the ambiguity associate with holding the asset.

## 2.2 *Intuition*

To provide some intuition with regard to the measure of ambiguity,  $\mathcal{U}^2[x] = 4 \text{Var}[P_L]$ , suggested by Izhakian (2011), lets consider the following binomial example. Assume an asset with the following two possible future returns  $d = -10\%$  and  $u = 20\%$ . Consider the case where the probabilities of  $d$  and  $u$  are known, say  $P(d) = P(u) = 0.5$ . The expected return is, thus, 5%. Taking the variance of outcomes, in terms of return, as a proxy for the degree of risk is 15%. Obviously, since the probabilities are precisely known, ambiguity is not present and the investors face only risk.

Assume now that, like in real life, the probabilities of  $d$  and  $u$  can be either  $P(d) = 0.4$  and  $P(u) = 0.6$  or alternatively  $P(d) = 0.6$  and  $P(u) = 0.4$ , where the two possible distributions are equally likely. That means that the investors are now facing not only risk but also ambiguity. The main idea of the measure of ambiguity,  $\mathcal{U}^2[\cdot]$ , is that, similar to measuring the degree of risk by the variance of outcomes, we can measure the degree of ambiguity by the variance of probabilities. However, concerning the variance of probabilities, we have to decide; the probability of which event the variance is applied. The natural choice would be the probability of the cumulative event of gain or the probability of the cumulative event of loss, for which the variance is identical since the event of loss is the complement of the event of gain and the probabilities are additive. Computing the variance of the probability of loss yields  $\text{Var}[P_L] = 0.01$  which in turn indicates a degree of ambiguity of  $\mathcal{U} = 0.2$ . Notice that the degree of risk has not changed since the variance is computed using the expected probabilities  $P(d) = P(u) = 0.5$ .

### 2.3 The risk-ambiguity model

Assume an economy in which the returns on all assets are normally distributed. The return on the market portfolio,  $r_m$ , is, therefore also normally distributed. The representative investor in this economy uses the risk free rate,  $r_f$ , as the reference point relative to which he classifies outcomes as a loss or a gain. That is, any return on the market portfolio lower than  $r_f$  is considered a loss and any return higher than  $r_f$  is considered a gain. Formally, the probability of loss takes the form

$$P_L = \Pr[r_m \leq r_f] = \int_{-\infty}^{r_f} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r_m - \mu)^2}{2\sigma^2}} dr_m = \Phi\left(\frac{r_f - \mu}{\sigma}\right), \quad (7)$$

where  $\Phi(\cdot)$  stands for the standard normal cumulative probability distribution. Recall, that in Shadow theory when ambiguity is present the variable  $P_L$  is random since the normal probability distribution is governed by the random parameters  $\mu$  and  $\sigma$ . We assume that these two parameters,  $\mu$  and  $\sigma$ , are uniformly distributed and that  $\mu$  is relatively close to  $r_f$ , which leads us to conclude that  $E[P_L] \approx E[P_G]$ . To allow tractability, and without loss of generality, the representative investor in our economy exhibits constant absolute risk attitude (CARA) and constant absolute ambiguity attitude (CAAA)<sup>9</sup>. The uncertainty premium, defined by Equation (3), is thus simplified to

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<sup>9</sup> Though we assume CRRA for risk, we assume CAAA for ambiguity. The literature usually documents CRRA for investors, see for example Kachelmeier, and Shehata.(1992), Chetty (2006), Schechter (2007) and Cohen and Einav (2007). CAAA means that the impact of the attitude toward ambiguity on the subjective probabilities of an event is decreasing with its expected probability. That is, the subjective probabilities of highly likely events are less affected by individuals' attitude toward ambiguity. Whereas, CAAA means that the impact of the attitude toward ambiguity on the subjective probabilities of an event is independent of its expected probability. We find that CAAA is more reasonable. Technically, the subjective probability of event  $j$  takes the

form  $Q_j = E[P_j] + \frac{1}{2} \frac{\psi''(E[P_j])}{\psi'(E[P_j])} \zeta_j^2$ , where  $E[P_j]$  is the expected probability of event  $j$ ,  $\frac{\psi''(E[P_j])}{\psi'(E[P_j])}$  is the coefficient of

ambiguity attitude and  $\zeta_j^2$  is the degree of event  $j$  measured by the variance of the probability, see Izhakian (2011). Therefore,

for CAAA, the subjective probability is  $Q_j = E[P_j] - \frac{\eta}{2E[P_j]} \zeta_j^2$  and for CAAA the subjective probability is

$$Q_j = E[P_j] - \frac{1}{2} \eta \zeta_j^2.$$

$$K = \gamma \frac{1}{2} \text{Var}[r_m] + \eta \frac{1}{4} \mathcal{U}^2[r_m], \quad (8)$$

where

$$\mathcal{U}^2[r_m] = 4 \text{Var} \left[ \Phi \left( \frac{r_f - \mu}{\sigma} \right) \right], \quad (9)$$

$\gamma$  is the coefficient of the investor's risk aversion and  $\eta$  is the coefficient of ambiguity aversion. A positive (negative)  $\gamma$  implies risk aversion (seeking), while a positive (negative)  $\eta$  implies ambiguity aversion (seeking). The expected return on the market portfolio,  $r_m$ , less the risk free rate, also called *excess return*, thus takes the form

$$E[r_m] - r_f = \gamma \frac{1}{2} \text{Var}[r_m] + \eta \frac{1}{4} \mathcal{U}^2[r_m], \quad (10)$$

where the risk premium is  $R = \alpha \frac{1}{2} \text{Var}[r_m]$  and the ambiguity premium is  $A = \eta \frac{1}{4} \mathcal{U}^2[r_m]$ . The effect of *uncertainty* on the return is now represented by two terms; a risk term and an ambiguity term. Each is measured separately and has different effects on the excess return. In the next section we present the empirical tests of this model. We first provide the methodology we use to measure the variables, especially the ambiguity measure, and then we apply the model to empirical tests.

### 3 Data and Methodology

#### 3.1 Data

The main body of data used in the empirical research is the intraday trading data (prices and volumes) on the exchange-traded fund SPDR (Ticker: SPY) taken from the TAQ database.<sup>10</sup> The Standard & Poor's Depository Receipts (SPDR) is comprised of all the stocks in the Standard & Poor's 500 Index. The stocks in the SPDR have the same weights as in the index and it is designed to track the index, before expenses. The expense ratio is about 7-8 basis points and the bid-ask spread is 1-2 basis points. The quarterly dividends are added to the index every 3 months. It can be sold short like any other stock and short interest is sometimes as high as 50

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<sup>10</sup> The Trade And Quotes (TAQ) database; Wharton Research Data Services (WRDS).

percent. A typical volume for the SPDR is between 200- 300 million shares per day, which is the highest of any US stocks traded on any exchange.

We use the SPDR as a proxy for the market portfolio and not the S&P index itself since the SPDR trades continuously, while the index contains illiquid stocks and so its values are stale. The data covers the period from February 1993 to December 2010.<sup>11</sup> Monthly returns adjusted for dividends obtained from the CRSP database.<sup>12</sup> VIX values were obtained from the CBOE site, and the risk free rate from Ibbotson Associates.<sup>13</sup>

### **3.2 Methodology**

The first step in designing the empirical tests is to compute the time series values of the variables that will be used in the tests. We first compute the degree of ambiguity derived by Izhakian (2011, Equation (9)) for each period of one month.<sup>14</sup> We sample the prices of SPY every 15 minutes starting from 9:30 until 16:00 each day: 27 prices in total for each day.<sup>15,16</sup> In case there was no trade at a specific sampling time, we took the volume weighted average value of the closest trading prices. Using these prices we compute 15 minute returns, 26 returns for each day.<sup>17,18</sup>

For each day we used its 26 observations to compute the mean and the variance of return. Depending on the number of trading days in the month, we have, for each month, between 440

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<sup>11</sup> Under the ticker symbol, SPY, SPDRs began trading on the American Stock Exchange (AMEX) on January 29, 1993.

<sup>12</sup> Since dividends are added to SPDR every three months, we adjust the return on SPDR, the explanatory variable, to monthly dividend yields, using the dividend yields on the S&P-500 index, taken from the CRSP database.

<sup>13</sup> The risk free rate is one-month Treasury bill rate of return (from Ibbotson Associates).

<sup>14</sup> For simplicity we concentrate on one month intervals, however the same procedure can be applied for periods of less than one month, 10 trading days for example.

<sup>15</sup> We also test our model using a 10 minutes interval; the results were essentially the same.

<sup>16</sup> To check for robustness, while eliminating the impact of the trading noise caused by opening and closing daily positions during first and the last half-hour of the stock trading, we also performed our tests using only the prices from 10:00 to 15:30. The results were essentially the same.

<sup>17</sup> We have not included returns between closing prices and opening prices of the following day. We eliminated the impact of overnight price changes and dividend distributions.

<sup>18</sup> While omitting the first and last half an hour of the trading our results remain almost similar.

and 572 observations.<sup>19</sup> Using Equation (7) we compute for each day the probability to suffer a loss,  $P_L$ . For each month, there is a vector which consists of 20 to 22 different loss-probabilities. Using this vector of loss-probabilities we compute its variance to obtain the **degree** of ambiguity,  $\mathcal{U}^2$ , for that month. Using the variances of each day over the month, we also calculate the mean of the variance,  $MVAR$ , and the variance of variance  $VVAR$  for that month. As controls, for each day we also compute the skewness and the kurtosis, and for each month the average skewness ( $MSKW$ ) and the average kurtosis ( $MKRT$ ). Using these variables, we next test the risk and ambiguity effect on returns, i.e., Equation (10) is subjected to regression tests presented in the next section.

### 3.3 *Descriptive Statistics*

The dependent variable is the monthly return on SPY, which serves as a proxy for the return on the market portfolio,  $r_m$ , minus the risk free rate,  $r_f$ , which is the 1 month T-bill rate. The market return,  $r_m$ , is computed using the opening price at the first trading day of the month and the closing price at the last trading day of that month.

The other variables that we use in the empirical tests are as follows:

*MVAR* – mean of daily variance. The daily variance is computed every day using 15 minutes rates of return (ROR) and multiplied by 26 (the number of 15 minutes intervals). It is averaged over the number of trading days in a month.

*VVAR* – variance of daily variance. The variance is computed by using the daily variances during the month.

*MSKW* – mean of daily skewness. The daily skewness is computed every day using 15 minute ROR, multiplied by  $26^{3/2}$  and averaged over the month.

*MKRT* – mean of daily kurtosis. The daily kurtosis is calculated every day using 15 minute ROR multiplied by  $26^2$  and averaged over the month.

*MVIX*<sup>2</sup> – mean of all daily squared VIX observation during the month.

*VVIX*<sup>2</sup> – variance of all daily squared VIX observation during the month.

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<sup>19</sup> To check for robustness, we formed randomly (without repetition) groups of 26 observations and computed a mean and a variance for each group. Since the results of this method were not significantly different from the first method, we conducted our tests using the first method.

$CVAR$  – variance of 15 minutes return of the last day in the month (converted to daily).

$CVIX^2$  – closing squared VIX on the last day of the month (converted to daily).

$DVAR$  – the change in variance from the last day in month  $t-1$  to last day in month  $t$ .

$DVIX^2$  – the change in squared VIX from the last day in month  $t-1$  to last day in month  $t$ .

Table I panel A provides summary statistics of the variables that are used in the empirical tests. All variables are adjusted to daily terms. During the 1993-2010 period, the daily mean return on SPY,  $r_m$ , is 0.025%, about 9.2 percent on an annual basis. The variance of  $r_m$  is about 3.0E-06. The risk free rate,  $r_f$ , is 0.009%, about 3.33 percent annually. The excess return,  $r_m - r_f$ , is 0.016%, 5.84 percent annually. The distribution of  $r_m - r_f$  is somewhat negatively skewed. Most values, however, are to the right of the mean. The positive kurtosis, 0.995, is an indication of fat tails.

The average daily variance (across all 215 months),  $MVAR$ , is about a half of the average daily  $VIX^2$  (0.00012 vs. 0.0002), while the variance is about the same (4.14E-08 vs. 4.00E-08). The average daily standard deviation is 0.885 percent while the average daily VIX is about 1.3 percent (or 14 percent vs. 21 percent on an annual basis). In the same vein, the average variance of the daily variance,  $VVAR$ , is about 8.11E-06 percent while the average variance of the daily VIX is only about 4.65E-09. Thus, VIX is on the average larger than the realized volatility by about 6-7 percent, but it moves in a narrower range.<sup>20</sup>  $MSKW$  is the mean of the daily skewness, which is negative but not significant.  $MKRT$  is the mean of the daily kurtosis, which is significantly different from zero, indicating fat tails as observed earlier.

Panel B in Table I provides summary statistics of the ambiguity parameters. The average expected daily probability of loss,  $E[P_L]$ , is 49.7 percent.  $E[P_L]$  is computed using Equation(4). The distribution of these probabilities is positively skewed, 0.054, and has thin tails (the kurtosis is -0.596).

The measure of ambiguity,  $\mathcal{U}^2$ , given in Equation (9), is on the average 0.025, where the minimum and maximum observed values are 0.0088 and 0.0532 respectively.  $\mathcal{U}$  (the square root

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<sup>20</sup> The variance of  $MVAR$  is the variance of the monthly average variance calculated for each day separately. The variance of the parameter  $VVAR$  is the variance of the monthly variance of the intraday variance.

of the measure  $\bar{U}^2$ ) is the daily ambiguity level measured in percentage points, 15.6. Figure 3 depicts the distribution of  $\bar{U}$  in the period between 1993 and 2010. The distribution of  $\bar{U}$  is almost symmetric around its mean of 16%.

Since our model predicts that the excess return should be affected by the ambiguity measure  $\bar{U}^2$  in a linear manner, we use  $\bar{U}^2$ , rather than  $\bar{U}$ , in our tests. Table I, panel B, shows that  $\bar{U}^2$  is positively skewed, with coefficient 0.6832, which is highly significant. Recall that the ambiguity measure takes on only positive values in the range between 0 and 1. The positive skewness thus indicates that the ambiguity level is usually concentrated around the mean with a long tail, where in some months we observe a relatively high level of ambiguity. The kurtosis of  $\bar{U}^2$  is slightly positive, 0.2437.<sup>21</sup>

It is important to note that the level of ambiguity can take on values between 0 and 1. It will be zero when the probability distribution is perfectly known. The level of ambiguity will be one, when we are faced with a lottery with two states of nature with the equally likely possible probabilities 0 and 1. In this case the probability of loss attains its maximum variation, which results in the highest possible volatility of probability, which in turn implies the highest possible degree of ambiguity. To get a more intuitive feel for the measure of ambiguity we can look at  $\bar{U}$  (not  $\bar{U}^2$ ). During the period 1993 to 2010, the mean level of ambiguity,  $\bar{U}$ , is about 15.6 percent, while its standard deviation is about 2.5%. The lowest recorded level of ambiguity is 9.3% and the highest is 23.1% (December 2002, when US decides to invade Iraq).

In Panel C, of table I, we provide the summary statistics of  $CVAR$ ,  $CVIX^2$ ,  $DVAR$ ,  $DVIX^2$ .  $CVIX^2$ , the value of  $VIX^2$  on the last day of the month is practically the same as  $MVIX^2$  but the variance is lower since  $CVIX^2$  does not include some extremely large observations. The same is true for  $CVAR$  and  $MVAR$ .

Table II provides the first 6 autocorrelation coefficients, of all the variables that we use in our tests. The variables that have large and significant autocorrelations are those who use VIX in their various forms, except for  $DVIX^2$ . For example,  $MVIX^2$  has a 1<sup>st</sup> order autocorrelation of 0.85

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<sup>21</sup> Since we also test the case of constant relative ambiguity aversion (CRAA), for completeness, the summary statistics of the normalized, relative ambiguity measure,  $\bar{U}/E[P_t]$  are also presented in Table I. One can see the relative ambiguity is positively skewed, 12.5951, but with negative kurtosis, -0.4084, which indicates thin tails.

and it decays slowly to 0.29 at the 6<sup>th</sup> order. A similar pattern is observed for  $VVIX^2$  and  $CVIX^2$ . The main concern is how it may affect the OLS estimator in our regression tests. We therefore conducted the tests proposed by Amihud and Hurvich (2004).

Figure 1 describes the average daily excess returns on the SPDR (SPY) over the years 1993 to 2010. Over this period we observe only a couple of months that contain big downward moves in the market. The two obvious ones, are in September of 1998, the Russian default and the LTCM debacle, and September 2008, the recent financial crisis. In Figure 2 we present the monthly ambiguity on a daily basis. It seems that during the 1990s ambiguity levels were not very high, but they have increased by at least 50% after 2000. It can be observed that relatively low returns are accompanied by relatively high levels of ambiguity in the previous month. For example, on August 1998 the excess return dropped to -0.7%, a month before, July 1998, the ambiguity level,  $\bar{U}^2$ , jumped to 0.0348. Or in August 2002, the return on the SPDR dropped to -0.52% the level of ambiguity in months before jumped up to 0.045. On September 2008 the return on SPDR dropped to the low -0.8%, where in months before ambiguity jumped to a level higher than 0.09.

[[ INSERT FIGURE 1 ]]

[[ INSERT FIGURE 2 ]]

Figure 3 describes the distribution of the degree of ambiguity,  $\bar{U}$ , in the period between February 1993 to December 2010. The degree of ambiguity is provided on the x-axis in percentages. The y-axis describes the frequency of the degree of ambiguity. Most of the observations are centered between 12% and 20% ambiguity. There are a few cases where the degree of ambiguity is higher than 20% or lower than 10%, which is actually very rare.

[[ INSERT FIGURE 3 ]]

Table III provides the cross correlations of all variables used in this study. It provides us with a first look at the relationships between all possible pairs of variables. In particular, the relationship of the excess return,  $r_m - r_f$ , and the ambiguity measure  $\bar{U}^2$ . It is negatively correlated and significant, which indicates that the coefficient of ambiguity is possibly negative. Or, in other words, that investors love ambiguity. Also, the ambiguity measure exhibits a low correlation with the other variables (though some are significant on the order of about 0.30). This basically rules out the possibility that ambiguity is a proxy for volatility or kurtosis. The various measures of volatility exhibit some correlation but those are not large enough to affect our main tests.

#### **4 Empirical Results: Testing the Effect of Ambiguity and Risk on Excess Returns**

Theoretically, the effect of ambiguity and risk on the expected excess returns, presented in Equation (10), assumes that investor exhibit constant relative risk attitude (CRRA) and constant absolute ambiguity attitude (CAAA).<sup>22</sup> In table IV we present the results of the regression tests where the dependent variable is the excess return and ambiguity and risk are the independent variable. We assume that the observed excess return is the best estimate of the expected excess return and so it is for the other variables like the expected risk measured by the daily variance. We have also introduced some variables, like volatility of volatility as a control variable.

In table IV the independent variables are measured contemporaneously with the excess return. So the return in month  $t$  is explained by the ambiguity in month  $t$ , by the variance in month  $t$ , etc. The values of the Durbin-Watson (DW) test indicate that we don't have a serial correlation issue. We first used only ambiguity to explain the excess return and found that the ambiguity effect is negative and highly significant. We then included  $MVAR$  and  $VVAR$ . Though the  $R^2$  has increased from 5% to 18%, we were puzzled by the sign and significance of the coefficients, which were negative for  $MVAR$  and positive for  $VVAR$ . We have expected the opposite, that the variance,  $MVAR$ , would be positive while  $VVAR$  would be negative. Since

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<sup>22</sup> We also tested our model for the case of constant relative ambiguity attitude (CRAA). The results were not significantly different than the results for the CAAA case.

these realized values may be poor proxies for expected variance and variance of variance, we then used  $MVIX^2$  and  $VVIX^2$  as our estimates of the expected risk measures. We now observe that  $MVIX^2$  is not significant but  $VVIX^2$  is negative and significant. We also included a measure of Kurtosis, denoted  $MKRT$ , to see whether the measure of ambiguity is possibly a proxy for Kurtosis, which turns out to be non significant and does not affect the significance of ambiguity.<sup>23</sup>

The results in table IV show that ambiguity is an important variable in explaining excess returns and is not a proxy for other possible factors. However, we did not find our measures of risk to have the effect dictated by our fundamental paradigm that implies a positive relationship between risk and return. In general, past empirical studies have not provided conclusive evidence, especially time-series tests. French, Schwert and Stambough (1987) is possibly the best known time series study that provides results that could be interpreted as supporting the basic theory, though they also come up with some mixed results. Since they only had a long time series of monthly data, we thought that the use of daily data (constructed from intraday data) may provide us with more promising results. In fact, our contribution is twofold. First, we argue that there is a missing variable, namely ambiguity. Second, we use data that are more fine-tuned to test the basic relationship between risk and return. In our tests we also argue that the measure of risk is orthogonal to the measure of ambiguity, which we observe in the low correlation of these two measures.

The results in table IV which use ex post measures of risk (and ambiguity), are consistent with the tests and results in other studies on the relationship between risk and return (e.g. French, Schwert and Satmbaough (1987)). In table V we use the measures of risk at  $t-1$ , coinciding with the market price at  $t-1$ . We argue that the level of risk and ambiguity at  $t-1$  affect the price at  $t-1$  and consequently the return from  $t-1$  to  $t$ . The  $t-1$  measures of risk and ambiguity could be considered ex ante (expected) estimates which should affect the return in time  $t$ . Since the explanatory variables are measured at  $t-1$ , it is the likely that we obtain biased coefficients and T-statistics due to a serial correlation. To deal with this issue, we subjected the regressions to the

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<sup>23</sup> We conducted the same test for skewness. We included a measure of skewness, denoted  $MSKW$ , to see whether the significance of the measure of ambiguity is affected by skewness. It turns out that skewness is not significant and does not affect the significance of ambiguity.

Amihud and Hurvitch (2004) test and all the regressions have “passed” the test, so no adjustments were necessary.<sup>24</sup>

The results in table V are encouraging. In all the regressions ambiguity is highly significant, while the risk measures have positive coefficients, though most of them are not significant. To measure risk in  $t-1$  we have used  $MVAR$ , the mean daily variance and  $MVIX^2$ , the mean daily  $VIX^2$ . To better align the measure of risk with the price at  $t-1$ , we have also used  $CVAR$ , the estimate of the variance on the last day of the month, adjacent to the opening price used for the return from  $t-1$  to  $t$ . Similarly, the last regression on table V uses the closing  $VIX^2$ , just before the opening price the next day. The best result is obtained when we include  $VVIX^2$ , the volatility of  $VIX^2$ , in the regression. Ambiguity is not affected, it is negative and as significant as in the other regressions. Expected volatility, estimated by  $MVIX^2$ , is positive with a coefficient of 2.6 and highly significant.  $VVIX^2$  has contributed to the results in two ways, it has increased the significance of the risk measure and has doubled the  $R^2$ , from about 6 percent to 12 percent. What role does  $VVIX^2$ , the volatility of volatility play? One possible explanation is that volatility of volatility is a proxy for liquidity where higher  $VVIX^2$  means higher liquidity. Thus, higher prices and lower returns.

Our next set of tests, provided in table VI, further support the findings in table V. As suggested by French, Schwert and Stambough (1987), we have used the unexpected change in risk to explain the excess return,  $r_m - r_f$ . We measure this change in two ways;  $DVAR_t$ , the change in the daily variance, from  $t-1$  to  $t$ , using the variance on the last day of the month.  $DVIX_t^2$ , the change in daily  $VIX^2$ , from  $t-1$  to  $t$ , using  $VIX^2$  on the last day of the month. The results in this table are our strongest results. We use the ambiguity measure,  $\mathcal{U}^2$ , in  $t-1$  to be consistent with the risk measures and can be considered an ex ante measure. As seen in the earlier tests, it is negative and highly significant. Both of the risk measures are also negative and

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<sup>24</sup> The Amihud and Hurvitch (2004) test is applied when the explanatory variable is a lagged variable. The residual from the OLS regression (the main model) is regressed against the residual from the autoregressive regression of the explanatory variable. According to this test, the estimated statistics are biased only if both of the following two conditions are satisfied. (i) the explanatory variables are highly autocorrelated. (ii) there is a statistically significant correlation between the residuals of the autoregressive regression of the explanatory variable and the residuals of the main regression, explaining  $r_m - r_f$  in our case. These two conditions have not been satisfied together in any of the regressions we tested.

highly significant as hypothesized. The regression which uses ambiguity and  $DVIX^2$  provides even stronger result, the  $R^2$  is about 46 percent. The ambiguity measure turns out to be significantly negative in any specification of the determinants of excess return. As stated in the introduction, previous evidences regarding the attitudes of ambiguity are mixed at best. Our results are consistent with the studies that show ambiguity loving. The effect of risk, measured by the unexpected change in volatility, can be interpreted as “indirect evidence of a positive ex ante relation” (see French, Schwert and Stambough (1987, p. 4)).

Examining the results in table V we see, for example, that in the regression with  $MVIX_{t-1}^2$  and  $VVIX_{t-1}^2$ , the coefficient of ambiguity  $\beta_1$  equals -0.0492 and the coefficient of risk ( $MVIX^2$ ),  $\beta_4$  equals 2.62. These results imply that the investor's coefficient of constant relative risk aversion is 5.24.<sup>25</sup> Though this number is in the range of estimates obtained in other studies (e.g. Brown and Gibbons (1985), French, Schwert and Stambough (1987)), it is on the high end indicating strong aversion to risk. The investors' coefficient of constant absolute ambiguity attitude, however, indicates that investors are typically ambiguity lovers characterized by coefficient of ambiguity loving equals to -0.2. To the best of our knowledge, this is the first empirical study which provides an estimate of the degree of the attitude towards ambiguity.

At first it seems puzzling that investors exhibit risk aversion and ambiguity loving at the same time. To explain this let's assume two assets with identical expected return, but the first asset has a random probability of loss/gain and the second asset's probability of loss/gain is equal to the expected probability of loss/gain of the first asset. By definition, an ambiguity lover prefers the first asset over the second asset. In our setting, returns are normally distributed, yet with random mean and random variance, such that if

$$E[\mathbb{P}(L | \mu, \sigma)] = E\left[\Phi\left(\frac{r_f - \mu}{\sigma}\right)\right] \leq \Phi\left(\frac{r_f - E[\mu]}{E[\sigma]}\right) = \mathbb{P}(L | E[\mu], E[\sigma]), \quad (11)$$

then the investors prefers the first asset with the random probabilities over the second asset with the constant probabilities. Since the returns on assets are assumed to be symmetrically distributed, a rational investor who maximizes expected return also minimizes the probability of

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<sup>25</sup> For completeness, we tested the impact of investors' *loss aversion* by controlling for different levels of risk aversion after facing a loss compared with the level of risk aversion after facing a gain. The results did not indicate a significantly different level of risk aversion for losses than for gains. That is, no evidences for loss-aversion preference were found.

loss. Thus, if inequality (11) holds a rational investor prefers the asset with the random probability. In other words, he must exhibit ambiguity loving.

Figure 4 provides a graphical representation of inequality (11). This figure assumes two possible normal probability distributions characterized by  $(\mu_1, \sigma_1)$  and  $(\mu_2, \sigma_2)$ . The y-axis depicts the probability of loss and the x-axis depicts the adjusted reference point (adjusted to the standard normal distribution). Given the random probabilities of loss, the expected probability of loss is  $E[P(L|\mu, \sigma)]$ . Assume now a second asset with constant mean,  $\mu = E[\mu_i] = \frac{\mu_1 + \mu_2}{2}$  and constant standard deviation,  $\sigma = E[\sigma_i] = \frac{\sigma_1 + \sigma_2}{2}$ . The probability of loss of this asset is  $P(L|E[\mu], E[\sigma])$ . Figure shows a case where the expected probability of loss is smaller than the probability of loss conditional on the expected mean and the expected variance, i.e.,  $E[P(L|\mu, \sigma)] < P(L|E[\mu], E[\sigma])$ .

[[ INSERT FIGURE 4 ]]

To check empirically that inequality (11) holds, for each month we compute monthly (i) the expected probability of loss assuming that the mean and variance governing the probability of loss are random and (ii) the probability of loss using the expected mean and expected variance in that month. The average expected probability of loss using (i) is 49.74%, while using (ii) the probability of loss is 50.17%. The difference between (i) and (ii) is negative (-0.43%) and significant ( $t = -2.06$ ). This result proves that the expected probability of loss when the parameters of the distribution are random, is lower than a constant probability of loss, using the expected parameters. A rational investor, who minimizes the expected probability of loss, prefers (i) over (ii) and therefore by definition he is an ambiguity lover.

Behavioral studies of decision making under ambiguity document that sometimes decision makers exhibit different attitudes toward ambiguity after facing a loss compared with the case where they face a gain (see for example, Bier and Connell (1994), and Chakravarty and Roy (2009a, 2009b)). Different attitudes toward ambiguity can be either different levels of ambiguity aversion/seeking or a change in attitude from ambiguity aversion to ambiguity

seeking. We also tested this hypothesis and found no evidences for different attitudes toward ambiguity. Ambiguity loving was observed for gains and for losses and the level of ambiguity loving after facing a loss was not significantly different than the level of ambiguity loving after facing a gain.

## **5 Conclusions**

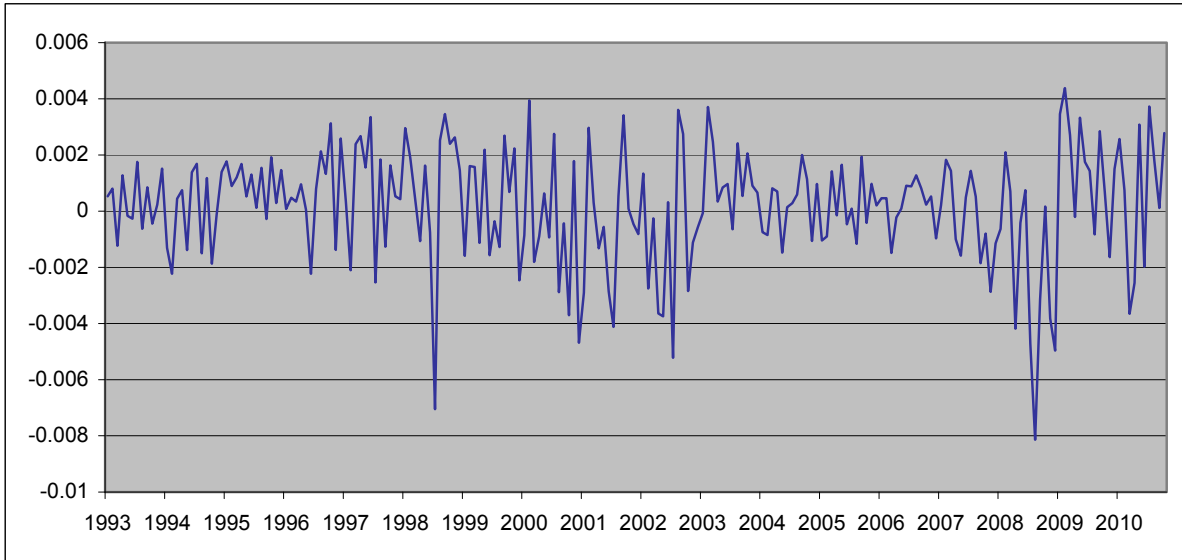
The basic tenet in asset pricing is the relationship between risk and return, which has been tested a multitude of times using a variety of models and factors. While this relationship could be tested on the market as a whole using time series data, most of these tests were cross-sectional. The results of these tests are mixed at best. In several studies the factor that measures the risk of the asset has a negative coefficient or is non significant while other factors (e.g. liquidity or liquidity risk) turn out to have the desired sign and are significant, which is a puzzle. One possibility is that the missing variable is ambiguity. In this study we introduce for the first time a measure of ambiguity, developed in Izhakian (2011). We use it in conjunction with measures of risk in time series tests. We claim that excess return on the market as a whole, known as the equity premium, is determined by two orthogonal factors; ambiguity and risk. We measure risk in a variety of ways, e.g., using rate of return variance and implied volatility. Our principle hypothesis is that both of the factors affect the excess return. While we expect, consistent with our asset pricing paradigm of risk aversion, that the measures of risk will be positively related to the excess return, we have no a-priori view of the effect of ambiguity. The results that we obtain are rather encouraging. The effect of ambiguity is negative and highly significant in all the tests that we employ. This is consistent with several recent studies that show that financial decision makers tend to be ambiguity loving. The effect of risk is generally positive, which is consistent with risk aversion but it's significance depends on the risk measure that we use. The best result that we obtain is when we use the unexpected change in volatility as the explanatory variable. Though this is an indirect test of the effect of risk on return, it provides the strongest evidence and is consistent with the results obtained by French, Schwert and Stambaghu (1987).

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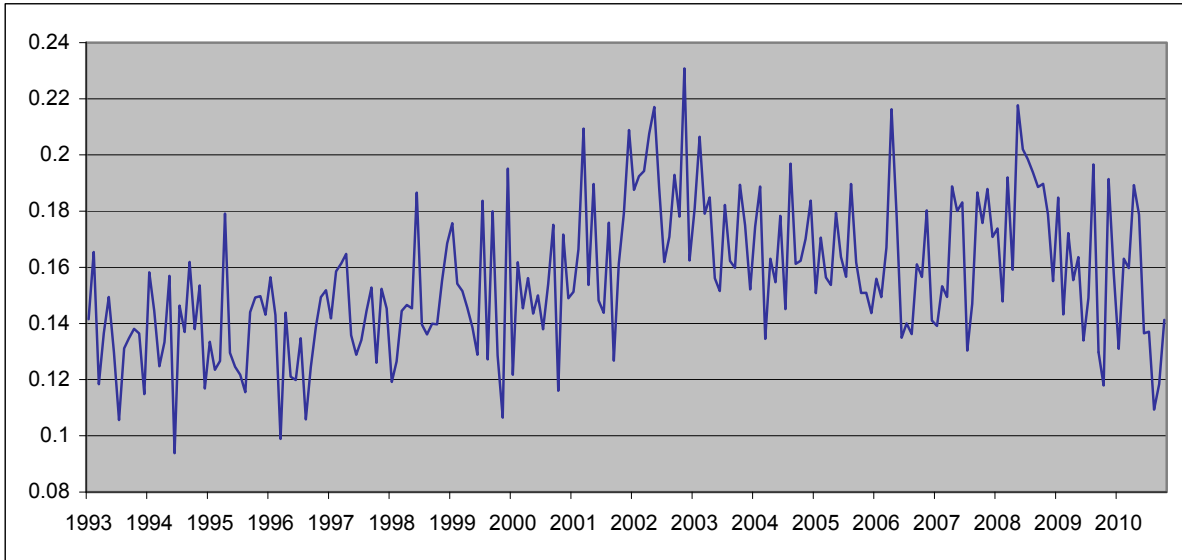
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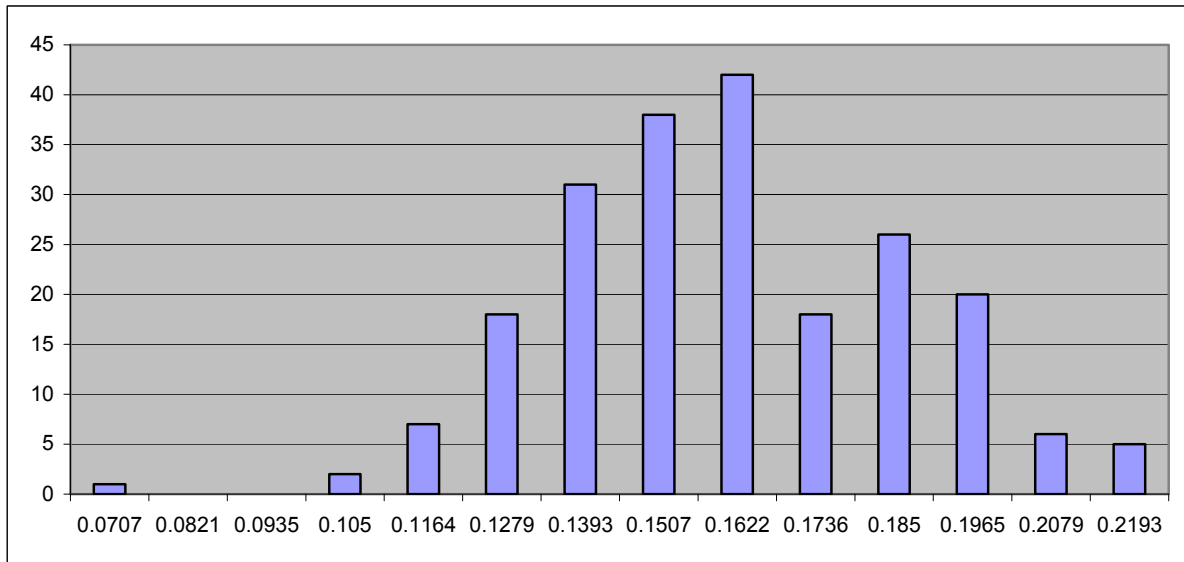
**Figure 1: Market excess return for the period 1993-2010**

This figure describes the daily adjusted to dividend excess return on the SPDR between February 1993 and December 2010. The values are the average daily excess return in each month.



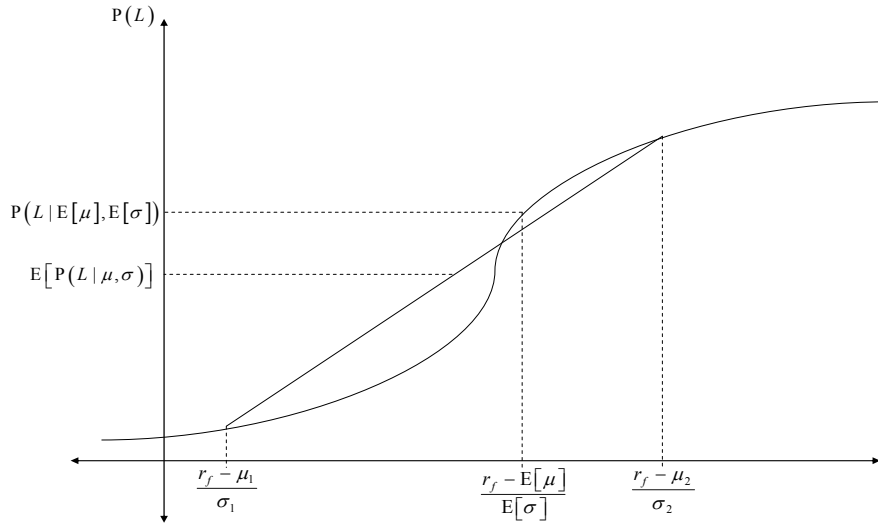
**Figure 2: The measure of absolute ambiguity level for the period 1993-2010.**

This figure describes the daily level of ambiguity, measured by  $\bar{U}$ , for each month between February 1993 and December 2010.  $\bar{U}$  is computed using 15 minutes rates of return during the month. For each day the probability of loss is computed using the mean and the variance of that day. For each month there are 20-22 probabilities of loss over which the standard deviation is computed to provide the squared degree of ambiguity,  $\bar{U}$ .



**Figure 3: The distribution of ambiguity level.**

This figure describes the distribution of the daily level of ambiguity measured by,  $\bar{U}$ , for the months between February 1993 and December 2010.  $\bar{U}$  is computed using 15 minutes rates of return during the month. For each day the probability of loss is computed using the mean and the variance of that day. For each month there are 20-22 probabilities of loss over which the standard deviation is computed to provide the squared degree of ambiguity,  $\bar{U}$ . Each column depicts the number of observations observed in the range describes on the x-axes.



**Figure 4: Ambiguity Loving**

This figure describes the probability of loss as a function of the threshold differentiating gains from losses, when to probability distribution is normal. The y-axis depicts the probability of loss and the x-axis depicts the value differentiating gains from losses. It assumes two possible normal probability distributions characterized by  $(\mu_1, \sigma_1)$  and  $(\mu_2, \sigma_2)$ . The expected portability of loss is  $E[P(L | \mu, \sigma)]$ . The probability of loss, when the mean and the variance,  $(E[\mu], E[\sigma])$  are the expected mean and the expected variance, respectively, of  $(\mu_1, \sigma_1)$  and  $(\mu_2, \sigma_2)$  is  $P(L | E[\mu], E[\sigma])$ .

**Table I****Summary Statistics of Regression Variables for the Period 1993-2010**

**Panel A** reports summary statistics for the entire sample between February 1993 and December 2010. All parameters are normalized to one day.  $r_m$  is the daily adjusted to dividend return on the SPDR.  $r_f$  is the daily return on the risk-free asset.  $MVAR$  is mean of daily variance. The variance is calculated every day using 15 minutes rates of return (ror) and multiplied by 26 (number of 15 minutes intervals). It is averaged over the number of trading days in a month.  $VVAR$  is variance of daily variance. The variance is calculated by using the daily variances during the month.  $MSKW$  is mean of daily skewness. The skewness is calculated every day using 15 minute ror, multiplied by 26 and averaged over the month.  $MKRT$  is the mean of daily kurtosis. The kurtosis is calculated every day using 15 minute ror multiplied by 26 and averaged over the month.  $MVIX^2$  is the mean of all daily VIX observation during the month.  $VVIX^2$  is the variance of all daily VIX observation during the month.

**Panel B** reports summary statistics for the measure of ambiguity.  $\bar{U}^2$  is the daily ambiguity level during the month,  $E[P_L]$  is the daily expected probability of loss and  $\bar{U}^2/E[P_L]$  is the normalized measure of ambiguity.

**Panel C** reports summary statistics for the volatility indicators.  $CVAR$  is the daily variance of the last trading day of the month.  $CVIX^2$  is the VIX observed in the last trading day of the month.  $DVIX^2$  is the difference of the observed VIX in the last trading day of the current month and the last trading day of the previous month.

**Panel A:**

	Mean	Variance	Skewness	Kurtosis	Min	Max	Median	N
$r_m$	0.000252	2.938E-06	-0.696084	1.018086	-0.005457	0.003368	0.000460	215
$r_f$	9.116E-05	2.870E-09	-0.314441	-1.264361	0.000000	0.000187	0.000103	215
$r_m - r_f$	0.000160	2.147E-06	-0.671609	0.995231	-0.005484	0.003364	0.000328	215
$MVAR$	0.000121	4.142E-08	5.777216	40.727858	1.660E-05	0.001787	6.925E-05	215
$VVAR$	8.115E-06	8.973E-09	14.298609	207.352920	1.11E-09	0.001378	3.86E-08	215
$MSKW$	-0.014659	0.0241834	0.124007	-0.487596	-0.365049	0.387104	-0.018444	215
$MKRT$	0.961168	0.5630023	1.433304	2.859667	-0.059868	4.707871	0.762767	215
$MVIX^2$	0.000200	4.007E-08	3.900617	20.981768	4.658E-05	0.001582	0.000158	215
$VVIX^2$	4.654E-09	5.459E-16	8.935092	87.099827	8.42E-12	2.64E-07	4.25E-10	215

**Panel B:**

$\bar{U}^2$	0.024988	6.606E-05	0.675753	0.240998	0.008802	0.053262	0.023582	215
$E[P_L]$	0.497510	0.0002281	0.050747	-0.587161	0.458340	0.538600	0.497787	215
$\bar{U}^2/E[P_L]$	0.050227	0.289621	13.316224	-0.410446	0.019203	0.098890	0.047374	215
$\bar{U}$	0.156041	0.0006425	0.274433	-0.221245	0.093817	0.230787	0.153565	215

**Panel C:**

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<i>CVAR</i>	0.000111	3.680E-08	8.754059	101.518093	5.53E-06	0.002436	6.481E-05	215
<i>CVIX</i> <sup>2</sup>	0.000198	3.319E-08	3.209620	14.729752	4.309E-05	0.001423	0.000154	215
<i>DVAR</i>	-1.457E-07	5.926E-08	0.892633	61.464498	-0.002097	0.002284	-3.00E-06	214
<i>DVIX</i> <sup>2</sup>	2.631E-07	1.232E-08	1.735123	19.411058	-0.000578	0.000808	-1.82E-06	214

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**Table II**  
**Autocorrelations**

This table reports the autocorrelations of all the different variables, explained and explanatory, which are used in the regressions. The autocorrelations are measured for the period between February 1993 and December 2010. All parameters are normalized to one day.  $r_m$  is the daily adjusted to dividend return on the SPDR.  $MVAR$  is mean of daily variance. The variance is calculated every day using 15 minutes rates of return (ror) and multiplied by 26 (number of 15 minutes intervals). It is averaged over the number of trading days in a month.  $VVAR$  is variance of daily variance. The variance is calculated by using the daily variances during the month.  $MSKW$  is mean of daily skweness. The skweness is calculated every day using 15 minute ror, multiplied by 26 and averaged over the month.  $MKRT$  is the mean of daily kurtosis. The kurtosis is calculated every day using 15 minute ror multiplied by 26 and averaged over the month.  $MVIX^2$  is the mean of all daily VIX observation during the month.  $VVIX^2$  is the variance of all daily VIX observation during the month.  $\mathcal{U}^2$  is the daily ambiguity level during the month,  $CVAR$  is the daily variance of the last trading day of the month.  $CVIX^2$  is the VIX observed in the last trading day of the month.  $DVIX^2$  is the difference of the observed VIX in the last trading day of the current month and the last trading day of the previous month.

	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$	$t-6$
$r_m - r_f$	0.0965	-0.0363	0.1213	0.0419	0.0442	-0.0481
$\mathcal{U}^2$	0.3836	0.3930	0.3777	0.2890	0.3499	0.3546
$MVAR$	0.4399	0.2375	0.2034	0.1837	0.0896	0.0376
$VVAR$	-0.0050	-0.0065	-0.0064	-0.0058	-0.0074	-0.0075
$MVIX^2$	0.8498	0.6424	0.5265	0.4601	0.3768	0.2859
$VVIX^2$	0.6542	0.3492	0.0998	0.0326	0.0226	-0.0117
$MSKW$	0.0262	0.1692	0.0525	-0.0388	0.1217	0.0506
$MKRT$	0.6299	0.5722	0.5702	0.5290	0.5338	0.5944
$CVAR$	0.1977	0.0834	0.0834	0.0839	0.0840	0.0806
$CVIX^2$	0.8149	0.5253	0.5253	0.4821	0.4123	0.3192
$DVAR$	-0.4326	-0.0033	-0.0033	0.0003	0.0021	0.0076
$DVIX^2$	0.0751	-0.0898	-0.0898	0.0725	0.0602	-0.1295

**Table III**  
**Cross Correlations of Variables**

This table reports the cross-correlations between the different variables, explained and explanatory, which are used in the regressions. The cross-correlations are measured for the period between February 1993 and December 2010. All parameters are normalized to one day.  $r_m$  is the daily adjusted to dividend return on the SPDR.  $MVAR$  is mean of daily variance. The variance is calculated every day using 15 minutes rates of return (ror) and multiplied by 26 (number of 15 minutes intervals). It is averaged over the number of trading days in a month.  $VVAR$  is variance of daily variance. The variance is calculated by using the daily variances during the month.  $MSKW$  is mean of daily skewness. The skewness is calculated every day using 15 minute ror, multiplied by 26 and averaged over the month.  $MKRT$  is the mean of daily kurtosis. The kurtosis is calculated every day using 15 minute ror multiplied by 26 and averaged over the month.  $MVIX^2$  is the mean of all daily VIX observation during the month.  $VVIX^2$  is the variance of all daily VIX observation during the month.  $\mathcal{U}^2$  is the daily ambiguity level during the month,  $E[P_L]$  is the daily expected probability of loss.  $CVAR$  is the daily variance of the last trading day of the month.  $CVIX^2$  is the VIX observed in the last trading day of the month.  $DVIX^2$  is the difference of the observed VIX in the last trading day of the current month and the last trading day of the previous month

**Panel A:**

	$r_m - r_f$	$\mathcal{U}^2$	$E[P_L]$	$MV$	$VV$	$MSKW$	$MKRT$	$MVIX^2$	$VVIX^2$
$r_m - r_f$	1.0000 – (0.0007)	-0.2294 (0.0007)	-0.7083 (<.0001)	-0.3499 (<.0001)	-0.0513 (0.4541)	-0.0336 (0.6239)	0.0770 (0.2611)	-0.2853 (<.0001)	-0.3093 (<.0001)
$\mathcal{U}^2$	-0.2294 (0.0007)	1.0000 – (0.0055)	0.1885 (0.0055)	0.2042 (0.0026)	0.0412 (0.5478)	0.0120 (0.8612)	-0.3183 (<.0001)	0.2778 (<.0001)	0.1983 (0.0035)
$E[P_L]$	-0.7083 (<.0001)	0.1885 (0.0055)	1.0000 – (0.0090)	0.1777 (0.0090)	0.0314 (0.6476)	0.0432 (0.5287)	-0.1261 (0.0651)	0.1719 (0.0116)	0.1025 (0.1340)
$MVAR$	-0.3499 (<.0001)	0.2042 (0.0026)	0.1777 (0.0090)	1.0000 – (<.0001)	0.6132 (<.0001)	0.1866 (0.0061)	-0.0949 (0.1656)	0.7339 (<.0001)	0.6926 (<.0001)
$VVAR$	-0.0513 (0.4541)	0.0412 (0.5478)	0.0314 (0.6476)	0.6132 (<.0001)	1.0000 – (0.0124)	0.1702 (0.1511)	0.0983 (0.1511)	0.0629 (0.3589)	0.0269 (0.6948)
$MSKW$	-0.0336 (0.6239)	0.0120 (0.8612)	0.0432 (0.5287)	0.1866 (0.0061)	0.1702 (0.0124)	1.0000 – (0.4672)	0.0498 (0.4672)	0.1017 (0.1374)	0.0497 (0.4682)
$MKRT$	0.0770	-0.3183	-0.1261	-0.0949	0.0983	0.0498	1.0000	-0.2851	-0.0725

	(0.2611)	(<.0001)	(0.0651)	(0.1656)	(0.1511)	(0.4672)	–	(<.0001)	(0.2897)
$MVIX^2$	-0.2853	0.2778	0.1719	0.7339	0.0629	0.1017	-0.2851	1.0000	0.7978
	(<.0001)	(<.0001)	(0.0116)	(<.0001)	(0.3589)	(0.1374)	(<.0001)	–	(<.0001)
$VVIX^2$	-0.3093	0.1983	0.1025	0.6926	0.0269	0.0497	-0.0725	0.7978	1.0000
	(<.0001)	(0.0035)	(0.1340)	(<.0001)	(0.6948)	(0.4682)	(0.2897)	(<.0001)	–

**Panel B**

	$r_m - r_f$	$\mathcal{U}^2$	$CVAR_{t-1}$	$CVIX_{t-1}$	$DVAR$	$DVIX$
$r_m - r_f$	1.0000	-0.2294	0.0824	-0.0091	-0.3806	-0.6483
	–	(0.0007)	(0.2301)	(0.8951)	(<.0001)	(<.0001)
$\mathcal{U}^2$	-0.2294	1.0000	0.0125	0.2141	0.0552	0.1007
	(0.0007)	–	(0.8557)	(0.0016)	(0.4216)	(0.1421)
$CVAR_{t-1}$	0.0824	0.0125	1.0000	0.5515	-0.6332	-0.1535
	(0.2301)	(0.8557)	–	(<.0001)	(<.0001)	(0.0248)
$CVIX_{t-1}^2$	-0.0091	0.2141	0.5515	1.0000	-0.2182	-0.3054
	(0.8951)	(0.0016)	(<.0001)	–	(0.0013)	(<.0001)
$DVAR$	-0.3806	0.0552	-0.6332	-0.2182	1.0000	0.4796
	(<.0001)	(0.4216)	(<.0001)	(0.0013)	–	(<.0001)
$DVIX^2$	-0.6483	0.1007	-0.1535	-0.3054	0.4796	1.0000
	(<.0001)	(0.1421)	(0.0248)	(<.0001)	(<.0001)	–

**Table IV**  
**Contemporaneous**

This table presents the contemporaneous regressions, i.e. explaining the market return at time  $t$  by the explanatory variables characterized in time  $t$ . The regressions use data for period between February 1993 and December 2010. All parameters are normalized to one day. The explained variable,  $r_m$  is the daily adjusted to dividend return on the SPDR.  $MVAR$  is mean of daily variance. The variance is calculated every day using 15 minutes rates of return (ror) and multiplied by 26 (number of 15 minutes intervals). It is averaged over the number of trading days in a month.  $VVAR$  is variance of daily variance. The variance is calculated by using the daily variances during the month.  $MKRT$  is the mean of daily kurtosis. The kurtosis is calculated every day using 15 minute ror multiplied by 26 and averaged over the month.  $MVIX^2$  is the mean of all daily VIX observation during the month.  $VVIX^2$  is the variance of all daily VIX observation during the month.  $\mathcal{U}^2$  is the daily ambiguity level during the month.

$$r_t = \alpha + \beta_1 \mathcal{U}_t^2 + \beta_2 MVAR_t + \beta_3 VVAR_t + \beta_4 MVIX_t^2 + \beta_5 VVIX_t^2 + \beta_6 MKRT_t + \varepsilon_t$$

$\alpha$	$\mathcal{U}_t^2$	$MVAR_t$	$VVAR_t$	$MVIX_t^2$	$VVIX_t^2$	$MKRT_t$	$R^2$	$Adj R^2$	$DW$
0.0012 (3.7636)	-0.0413 (-3.4285)						0.0525	0.0481	1.9875
0.0012 (3.9130)	-0.0297 (-2.5404)	-2.2773 (-4.8742)					0.1484	0.1403	2.0892
0.0012 (4.0144)	-0.0259 (-2.2439)	-3.3785 (-5.7941)	3.7486 (3.0543)				0.1846	0.1730	2.0546
0.0012 (4.0325)	-0.0294 (-2.4035)			-1.7589 (-3.5442)			0.1058	0.0973	2.0135
0.0011 (3.3488)	-0.0304 (-2.5095)			-0.3703 (-0.4625)	-14769.4049 (-2.1984)		0.1259	0.1134	2.0903
0.0012 (2.9914)	-0.0411 (-3.2258)					0.0000 (0.0611)	0.0526	0.0436	1.9867

0.0013	-0.0281	-3.4286	3.8875			-0.0001	0.1863	0.1708	2.0684
(3.6004)	(-2.3340)	(-5.8327)	(3.1216)			(-0.6385)			
0.0011	-0.0309			-0.4103	-14510.4128	0.0000	0.1261	0.1094	2.0913
(2.6626)	(-2.4615)			(-0.4828)	(-2.0827)	(-0.1423)			

**Table V**  
**Prediction**

This table presents the predictive regressions, i.e. explaining the market return at time  $t$  by the explanatory variables characterized in time  $t-1$ . The regressions use data for period between February 1993 and December 2010. All parameters are normalized to one day.  $r_m$  is the daily adjusted to dividend return on the SPDR.  $MVAR$  is mean of daily variance. The variance is calculated every day using 15 minutes rates of return (ror) and multiplied by 26 (number of 15 minutes intervals). It is averaged over the number of trading days in a month.  $VVAR$  is variance of daily variance. The variance is calculated by using the daily variances during the month.  $MSKW$  is mean of daily skewness. The skewness is calculated every day using 15 minute ror, multiplied by 26 and averaged over the month.  $MKRT$  is the mean of daily kurtosis. The kurtosis is calculated every day using 15 minute ror multiplied by 26 and averaged over the month.  $MVIX^2$  is the mean of all daily VIX observation during the month.  $VVIX^2$  is the variance of all daily VIX observation during the month.  $\bar{U}^2$  is the daily ambiguity level during the month.  $CVAR$  is the daily variance of the last trading day of the month.  $CVIX^2$  is the VIX observed in the last trading day of the month.

$$r_t = \alpha + \beta_1 \bar{U}_{t-1}^2 + \beta_2 MVAR_{t-1} + \beta_3 VVAR_{t-1} + \beta_4 MVIX_{t-1}^2 + \beta_5 VVIX_{t-1}^2 + \beta_6 CVAR_{t-1} + \beta_7 CVIX_{t-1}^2 + \varepsilon_t$$

$\alpha$	$\bar{U}_{t-1}^2$	$MVAR_{t-1}$	$VVAR_{t-1}$	$MVIX_{t-1}^2$	$VVIX_{t-1}^2$	$CVAR_{t-1}$	$CVIX_{t-1}^2$	$R^2$	$Adj R^2$	$DW$
0.0013 (4.0805)	-0.0451 (-3.7589)							0.0625	0.0581	1.9385
0.0013 (4.0798)	-0.0466 (-3.8004)	0.3022 (0.6167)						0.0642	0.0553	1.9069
0.0013 (4.0854)	-0.0455 (-3.6806)	-0.0396 (-0.0635)	1.1629 (0.8860)					0.0677	0.0543	1.9068
0.0013 (4.0379)	-0.0475 (-3.7991)			0.3513 (0.6923)				0.0646	0.0557	1.9256
0.0010 (3.0691)	-0.0492 (-4.0437)			2.6214 (3.2638)	-24163.5678 (-3.5833)			0.1185	0.1059	2.0224
0.0012 (3.9160)	-0.0466 (-3.8864)					0.7915 (1.5573)		0.0731	0.0644	1.8958
0.0013 (3.9967)	-0.0483 (-3.8708)						0.5225 (0.9383)	0.0664	0.0575	1.9109

**Table VI**  
**Changes Impact**

This table presents the changes regressions, i.e. explaining the market return at time  $t$  by the changes in the explanatory variables between time  $t-2$  and time  $t-1$ . The regressions use data for period between February 1993 and December 2010. All parameters are normalized to one day.  $r_m$  is the daily adjusted to dividend return on the SPDR.  $\mathcal{U}^2$  is the daily ambiguity level during the month.  $DVAR$  is the difference of the observed VAR in the last trading day of the current month and the last trading day of the previous month.  $DVIX^2$  is the difference of the observed VIX in the last trading day of the current month and the last trading day of the previous month.

$$r_{m,t} = \alpha + \beta_1 \mathcal{U}_{t-1}^2 + \beta_2 DVAR_t + \beta_3 DVIX_t^2 + \varepsilon_t.$$

$\alpha$	$\mathcal{U}_{t-1}^2$	$DVAR_t$	$DVIX_t^2$	$R^2$	$Adj R^2$	$DW$
0.0012 (3.9799)	-0.0404 (-3.6142)	-2.1990 (-5.8852)		0.1947	0.1870	1.9517
0.0011 (4.6313)	-0.0379 (-4.1587)		-8.4014 (-12.5774)	0.4642	0.4591	1.8360
0.0002 (1.7097)		-2.2957 (-5.9918)		0.1448	0.1408	1.8091
0.0002 (2.1103)			-8.5763 (-12.3971)	0.4203	0.4175	1.7145